Remembering Math:

The Design of Digital Learning Objects to Spark Professional Learning

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1.0 Introduction

This paper describes the design of digital learning objects to spark professional learning. Our design challenge was to build learning objects[1] that would help experienced special education teachers, who had been teaching in math classes, to demonstrate their proficiency in middle and secondary school mathematics on the PRAXIS II[2] exam, a standardized assessment used for teacher certification and licensure. Exams such as the PRAXIS II act as a certification gateway for teaching in subjects in which the teacher is not certified (NMAP, 2008). For many of the teachers involved, taking the PRAXIS II exam did not involve learning new content. Instead, the teachers were faced with remembering knowledge they had once learned but had now forgotten how to access and apply. In order to pass the test, teachers could either retake the appropriate content courses (CBMS, 2001) such as those designed to teach new content about how to acquire, organize and deliver subject matter content (Borko & Putnam 1996; Fennema & Franke, 1992; RAND Mathematics Study Panel Report, 2002), or they could take test-preparation courses to help them game the system. Requiring experienced teachers to take new undergraduate or remedial math coursework is impractical, and training teachers to "game" the test process is certainly not consistent with aim of the certification policy, and borders on unethical. Our task was to use learning object design to develop a solution that incorporates both approaches– a technological solution to help teachers remember and correctly use the math they once learned.

Our approach to design is based on the insight that a central aspect for helping students to remember content they once learned is to uncover areas of procedural

[1] We use the term *digital learning object* to refer to reusable tools designed to assist the learner in understanding smaller kernels of knowledge than those employed in traditional curricula.
[2] The PRAXIS II exam is published by the Educational Testing Service (ETS) and is widely used for teacher licensure and certification. For more information, see: http://www.ets.org/

breakdown in their attempts to solve math problems. Remembering math involves reassembling misplaced, broken or fragmented conceptual knowledge once learned in school. The design of learning objects allowed us to determine which aspects of PRAXIS II-type questions highlight conceptual breakdown, and lead us to build learning objects that would help learners reassemble prior concepts to improve capacity to solve similar problems. Our paper reports on a design-based research investigation to build, implement and assess a series of math learning objects for adult learners. Over the course of 2 years, we built 12 web-based learning objects, and tested them with 59 adult learners. We found that they produced significant learning gains with their intended audience.

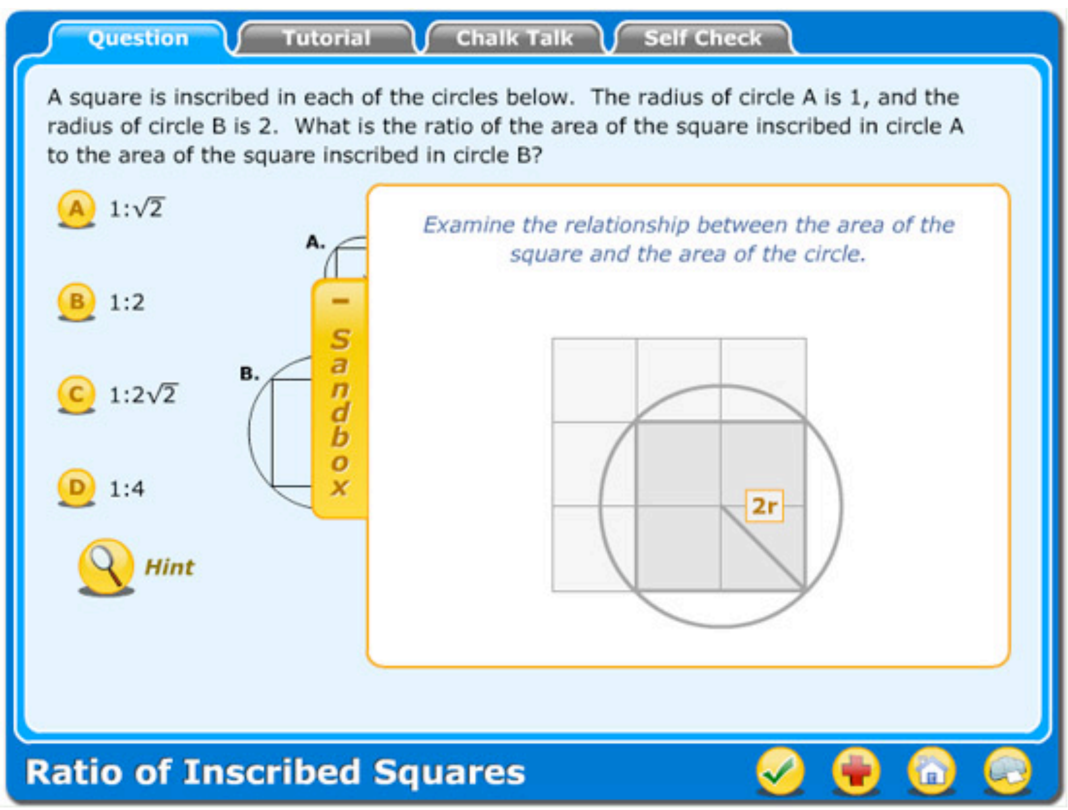

Figure 1: Sample RMLO Structure

Our paper offers an account of how our theory of professional learning led to a design process that resulted in the development of the Remembering Math Learning Objects (RMLO) (Figure 1)[3]. The first section of the paper recaps how our thinking about the learning requirements for the objects, grounded in cognitive theory and human-computer interaction research, led us to a design strategy and format for the objects. We describe how our collaborative design process was structured to elicit the breakdown points present in typical math problems and how this process played out in the RMLO design. In the next section of the paper, we discuss the RMLO format and function. Finally, we describe an evaluation process that reports the learning gains made by members of our target audience. We hope that the design process we describe will help shift research attention towards principled efforts to articulate how to ground the design process in a clear theory of action.

2.0 Designing a Design Process for Remembering Math

The Remembering Math project is guided by a theory of action for how to spark adult recollection of previously learned content. Our interest was to develop learning objects that would help learners revive knowledge about appropriate procedures in situations that called for certain kinds of mathematical problem solving. This is not to suggest all mathematical thinking involves the recall of procedural knowledge – math learning research clearly shows that mathematical reasoning is much more that recalling and applying appropriate formulae (c.f. NRC, 2001). We would certainly agree that, even in this case, a comprehensive approach to teacher education in math would reach far beyond recall to encompass pedagogical content knowledge, learning from best practice

[3] The *Remembering Math Learning Objects* described in this paper are available at http://www.slg.gameslearningsociety.org/cesa_files/learningobjects.php#

examples of teaching, and understanding how to diagnose typical student learning patterns (Ball, Hill & Bass, 2005). Our aim here is much more modest and targeted toward the practical task of helping teachers recall what they need to pass content exams. The modesty of our design goals allows us to focus on the learning requirements of the specific task at hand, which in turn provides the kinds of specific connections between theory and design that can actually inform practical action. Our theory of action is based on answers to two questions:

- How do adults forget previously learned math knowledge?
- Are there common patterns of recall that obstruct appropriate use of once-learned math knowledge?

In this section we provide some theoretical grounding for each of these steps in our theory of action, then describe how we organized and conducted a design process to develop the learning objects.

2.1 How do adults forget math knowledge?

Our approach to understanding forgetting is grounded in theories of how people learn and appropriately apply procedural knowledge. Math content is typically learned in schools as a collection of procedures that need to be accurately applied to produce correct answers. Learners are judged to know the content if they can select and apply the appropriate procedure (as a formula) at the appropriate time to produce an appropriate answer. Homework is designed to allow the learner to repeat the procedure, with slight variations in application, a sufficient number of times so that the link between the rule and its application becomes a matter of habit. The emphasis on developing and applying procedural knowledge in the teaching of mathematics has long been criticized by math educators and reformers (c.f. Hill, Rowan & Ball, 2005; Ma, 1999). Reformers argue that emphasizing procedural knowing prohibits the development of concepts that serve to create mathematical fluency (Fuson, Kalchman, & Bransford, 2004).

Schema theory as a descriptive model of forgetting

The frequent consequence of a procedural approach to learning math is long-term inability to remember the procedural steps or to forget the rules that govern the correct application of the procedures. Cognitive scientists have proposed schema theory to explain how information and procedures are organized into enduring, accessible capacities. Schemas link together facts, causal explanations, stories and procedures so that people can act and make predictions about the world (Murphy & Medin, 1985; Schank, 1986). Schemas also reflect the social contexts in which they were developed (Cantor, Mischel, & Schwartz, 1982). The knowledge embedded in a schema is activated either through top-down or bottom-up cues (Rumelhart, 1980). A top-down cue triggers the recall facts or procedures through exposure to a general concept; bottom-up cues enable the recall of concepts through an encounter with facts and procedures. From this perspective, a robust, working disciplinary understanding means developing a schema in which sequences of details are linked together and organized by general concepts, and concepts trigger activation of the appropriate procedures and facts.

Mathematical misunderstanding then, results from a disconnect between procedures and concepts or the inability to recall how steps in procedures hang together in ways that lead to the general concept. This approach to misunderstanding is readily illustrated in how adults recall the wrong procedure for a given problem, or are unable to recall how the procedure can be used to solve the problem. Such difficulties raise the question of how schemas actually break down and lead to inaccurate recall. One problem is the unreliability or irrelevance of the information stored in a schema. Schemas are seldom clean, independent structures – they typically include scripts, seemingly cross-linked at random, that refer to details of the situation in which the learning occurred, emotional reactions to the learning process, and mathematically irrelevant stories about people, places or contexts referenced in the schema (Schank, 1986). If the scripts that link concepts and procedures are not sufficiently rehearsed or explained, the whole schema proves an unreliable structure to facilitate accurate recall. This leads to the seemingly random, fragmented strategies and emotions that occur when learners try to recall appropriate solution procedures. The other kind of problem occurs when schemas get "filled in" with generic details drawn from related kinds of experiences. Learners can interpret problems in terms of what they expect the problem to be, and to ignore salient aspects of problems that challenge schematic interpretation (Kolodner 1993; Spillane, Reiser & Reimer, 2002). This leads learners to respond to "false cues" that trigger inappropriate solution paths because schemas distract learners from salient aspects of a problem.

Schema theory helps us see the state of mathematical knowledge in the mind of a typically educated adult who does not use formal mathematical procedures in daily practice. Like the remains of an ancient city, the landscape is littered with once-functional artifacts that now lie scattered about with little apparent connection to the surroundings. Many of the relevant pieces are still present, but important pieces are either misplaced or completely inaccessible. New development has obliterated once-significant reference points and has co-opted existing architectures into new orientations.

For our purposes, it is important to note that a) there was once an organized system of reference that drew the disjointed pieces into an operational whole, and b) that this former organization can often still be recovered, with the appropriate prompts, to serve the functions it was once learned to address.

2.2 Are there common patterns of recall to recover once-learned math knowledge?

Most people forget the details and procedures of the math they learned in elementary and secondary schools without subsequent opportunities to use their knowledge in the course of their lives. Designers are faced with an interesting challenge in their efforts to build evocative learning environments: are there patterns in the ways that people forget and remember math knowledge, or does each person forget and recall in their own way? In the 1990s, researchers interested in applying cognitive theories to math and science teaching focused on the phenomena of identifying common misconceptions (Smith, DiSessa & Rochelle, 1994). Researchers assumed that misconceptions are caused when learners use schema they already have in order to understand new schema. This mismatch of new and old conceptual schema can lead to a series of common mistakes. Misconception research assumed that the common mistakes learners make in mastering new material implied common features in the underlying processes of knowledge organization. To be sure, misconception researchers did not assume uniform underlying cognitive processes, but they did assume that much of the variation in student struggles with key math and science concepts could be better treated once researchers had uncovered the common misconceptions. Understanding typical errors in the learning process would provide teachers with guidelines for understanding and shoring up learners' conceptual structures in order to build more robust orthodox math and science schemas.

Misconception research typically seeks to understand how to "fix" prior learned schema in terms of the intended new schema (e.g. Hiebert and Behr, 1988; Brown & Clement, 1989). Our intention in the RMLO design was to decouple misconception from reform-based pedagogical aims. This decoupling would allow designers to elicit common misconceptions as paths to prior understanding, rather than as a condition for replacing prior understanding. If misconceptions reveal how learners typically misunderstand new knowledge, they should also help us see the patterns in how learners can recover prior knowledge. These patterns could then be used to structure activities that can anticipate how people typically snag on the features of a given math problem.

3.0 Remembering Math Design Process

In the field of computer interface design, John Carroll (1990; 2002) described a "minimalist design" based on insights about how people typically navigate an interface. A minimalist design approach is particularly relevant for the development of learning objects based on common misconceptions. The minimalist design process involves observing the intended learning process, noting where learner understanding typically breaks down, and developing just enough instructional support to allow learners to make the appropriate moves or inferences. Minimalist design emphasizes the following characteristics (Carroll, 1990):

1. Allow learners to start immediately on meaningful tasks;
2. Minimize the amount of reading and other passive forms of training by allowing users to fill in the gaps themselves;

3. Include error recognition and recovery activities in the instruction; and
4. Make all learning activities self-contained and independent of sequence.

Minimalist design stands in stark contrast to a standards-based instructional approach that seeks to specify all intended learning outcomes and behaviors for any learner. Standards-based learning aims for comprehensive approaches to teaching – minimalist design aims for the appropriate level of representation necessary to complete the given task. Finally, a minimalist approach to design requires considerable lead time for the designers to develop a process of observation and interaction that will reveal the appropriate opportunities for design. Standards-based learning environment designers can start right in on the design; minimalist designers must wait to understand just what needs to be designed.

Our goal was to use the minimalist design strategy to identify just where typical learner understanding would break down in solving traditional test problems. We began with a participatory design model (Schuler & Namioka, 1993; Shrader et. al., 1999) that would integrate the typical learner's perspective into the design process. A participatory design process can be easily adapted to meet minimalist design goals. Participatory design meetings can be used to engage learners in the intended learning process in order to identify procedural and conceptual breakdowns. Such a design process would begin with working toward a solution for a typical math problem while paying close attention to where understanding breakdowns occur. These breakdowns serve to identity occasions for learning object design. Once a prototype learning object is built, the team can repeat the design process to determine the degree to which the learning supports integrated into the object correspond to the occasions where understanding breaks down.

While the development of an instructional design template was by no means unprecedented, the process utilized in template design for the RMLOs served as a means to instantiate learning opportunities that accorded with the theoretical framework derived at the beginning of the project. By eliciting occasions for conceptual breakdown, the design process allowed us to target design at just the points where learners were most likely to struggle. Building the learning objects around these occasions allowed us to test whether the breakdowns were indeed the places where a wider range of adult learners would also struggle.

The design team first determined which problems would be best for the design of learning objects. We decided that anchoring the design process around specific problems would help us ground the solution paths in actual struggles with recollection. However, to avoid creating idiosyncratic learning objects, we agreed to work toward a common template for representation that would include the same kinds of prompts for each sample problem. In order to determine which problem types we would use for the design, we surveyed 75 adult learners to assess the difficulty of a range of sample math problems. We asked participants to rank each problem in terms of their confidence to solve the problem accurately. We then selected 12 problems that raters were least confident in their ability to solve (Appendix 1).[4]

3.1 Remembering Math Learning Objects

The Remembering Math learning objects were built to remind adult learners of the math they had once learned. The design process aimed toward two central goals: a) a template that would provide a common foundation for representing remembering cues, and b) the development of a design team that would elicit the learner's procedural and

[4] Each of these RMLO problems includes the "hint" (described on p. 19) developed by the design team.

conceptual breakdowns. The following section recounts the iterative development path toward these two design goals. First, the design team identified four distinct occasions for remembering. Second, the design team composition and interaction was structured to elicit opportunities for breakdown. Third, the team then fitted the identified opportunities for breakdown in the context of the design template.

Occasions for remembering. Before approaching the individual problems, the design team worked to create a template for the learning object that would offer a generic framework to identify where conceptual breakdown can occur for learners attempting to remember mathematical problems like those found on the PRAXIS II exam. This resulted in the identification of four discrete types of conceptual breakdown: trouble recalling how to start on a problem, trouble recognizing the pattern of a problem that allows for working through the intermediate steps, trouble remembering the appropriate sequence for problem solving, and inability to recognize or recall the problem type. Learning object users were presented with four levels of structured "reminding" that corresponded to the areas of conceptual breakdown: *hint*, *sandbox*, *tutorial* and *chalk talk*. Each "reminding function" used different forms of representation to address different aspects of the perceived conceptual breakdown in each math problem.

Design team composition and function. Ensuring optimal design team composition was perhaps the most significant factor in the design of the Remembering Math learning objects. Our team included three kinds of participants: graphic designers, content experts, and intelligent novices. Graphic designer involvement in the core development process served to keep the team aware of the range of content representations. Content experts were required to ensure that the procedures we built into the learning objects would have solid mathematical foundations. However, investigations into expertise have revealed that content experts are not necessarily effective at conveying their knowledge of content within their field, unless they have expertise in both the content area and pedagogical mechanisms for instruction (Bransford, et. al., 2000; Shulman, 1987). For this reason, we determined that it was essential to balance content expertise on our design team by retaining members whose mathematical understanding had broken down in the same ways as the learners for which the objects were designed.

If the team could not foresee potential learning breakdowns, it would be blind to the critical occasions for development. We thus decided to include several math "intelligent novices" to lead the development process. The concept of intelligent novice has been used in cognitive psychology to describe learners with good generic problem solving skills but weak domain content knowledge (e.g. Bruer, 1993). In our work, an intelligent novice is able to call into question those assumptions that the experts likely do not examine around what aspects of a specific worked example added explanatory value for our targeted learners. Just as with our intended audience, intelligent novice team members had the appropriate prior math instruction as well as a lapse between when they had received this instruction comparable to that of the practicing educators for whom the learning objects were being designed. Finally, the intelligent novices were of similar status to the content experts in order to prevent the content experts from setting the design agenda. If the intelligent novices had yielded the direction of the design to the content experts, the learning object design would have likely reflected how learners *ought* to learn, rather than how they actually *have* learned. The intelligent novices, therefore, directed the design process, relying on their candor to identify the essential confusions that arose in the problem-solving process of a given math problem. The identified confusion then served as the starting point for learning object design as the group sought representations that would make the occasion for conceptual breakdown explicit, then to provide tools for learners to draw upon pre-existing mathematical ideas to "reconstruct" a viable solution path. In addition these two team members were able to identify whether potential cues actually served as effective stimuli for helping learners to overcome conceptual breakdowns.

The design process for building each learning object involved a sequence of three basic steps: intelligent novice talk-aloud; math expert suggestions; and collaborative design of the tutorial (Figure 2). The design of each learning object was structured around a specific math problem. The team began by utilizing and exploring the parts in a conventional instructional sequence where the math novices seemed to encounter difficulties. We invited the intelligent novice design team members to talk their way through the solution processes until they got stuck on a specific definition,

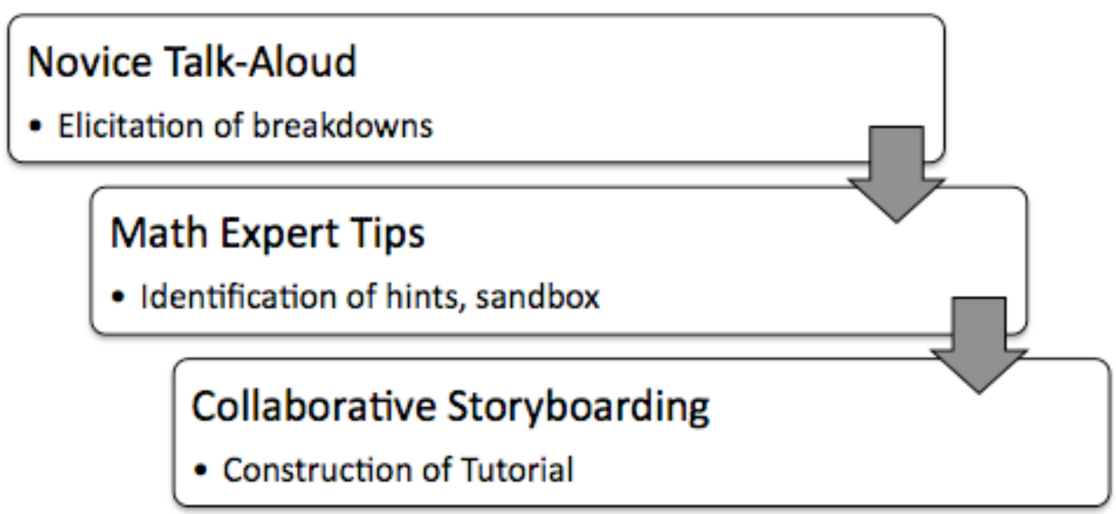

Figure 2: Learning Object Design Process

procedure or graphical feature of the problem. The math experts would then provide hints and definitions to clarify the initial breakdown. These suggestions formed the initial versions of the hints and of the variables that could be manipulated. After the experts and novices became familiar both with the problem and with the novice misunderstandings of the problem, the team would then use the graphic designer to storyboard a sequential representation of a problem-solving process that would directly address the identified bottlenecks. The graphic designer would then create a Flash-based representation of the storyboard as the first draft of the Tutorial; the critique and refinement of this first draft of the tutorial would then serve as the subject for the follow-up design meeting. Finally, after a variety of users were invited to use the prototype learning object, the design team would meet once again to revisit the initial assumptions about the identification of bottlenecks, and the representation of hints, sandbox and tutorial learning object elements.

Fitting occasions for remembering into the Learning Object template. The substantive design process involved fitting the identified conceptual and procedural breakdowns into the context of the Learning Object template. In the following sections we provide several illustrations of the functions and design of template features.

Hints (Figure 3): We designed hints in order to provide a cue for a learner who remembers the right procedure, but needs a salient clue to get over the initial misconception involved in choosing the right answer. Hints are designed to offer a direct suggestion about just where appropriate procedure has broken down. The design team relied on math content experts to determine the appropriate hint included for each problem. Typical hints included the reminders that math teachers would give students at

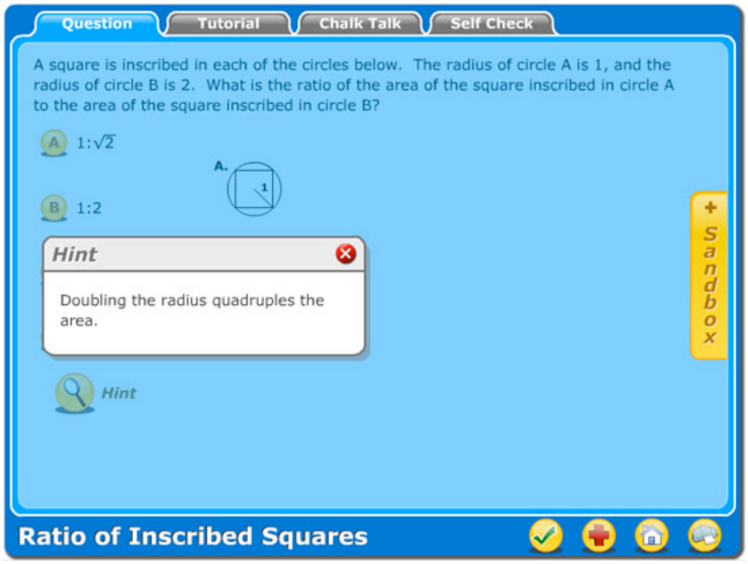

Figure 3: Hint Example

just the point where students would go astray. For example, in Learning Object Problem 8 (see Appendix 1), the math experts talked through how to help students to begin thinking about the probability that a coin would end up in a given square. The "problem-setting script" proposed by the math experts pointed out how students needed to begin with an estimate the probabilities of based on adjacent locations to a given square. Thus, reminding students "How many squares are adjacent to (a given location)?" might help students with a fuzzy idea about how to solve this type of problem with a better understanding of how set up the application of the probability calculation procedure.

Other hints required the intelligent novices on the design team to "unpack" the seemingly seamless chain of reasoning math experts used to apply the appropriate procedure. For example, in the RMLO 10 (Stem and Leaf plot), math content experts jumped right into the solution path while the intelligent novices were not able to remember how a stem-and-leaf plot worked. The design team reasoned that Learning Object users would likely not have rich, flexible math understanding, and that a significant confusion in the way the problem was presented would likely thwart efforts to recover prior reasoning about this kind of problem. The hint: "Each leaf represents the weight of one member" was intended to remind the learners about how the graphic worked so that they could get to the issue of thinking though the variables presented in the problem. Hints were selected to clear up the kinds of conceptual or graphical confusion (that test-based story problems seem to specialize in!) and allow learners who had a vague recollection about the appropriate solution path focus on recalling what they knew.

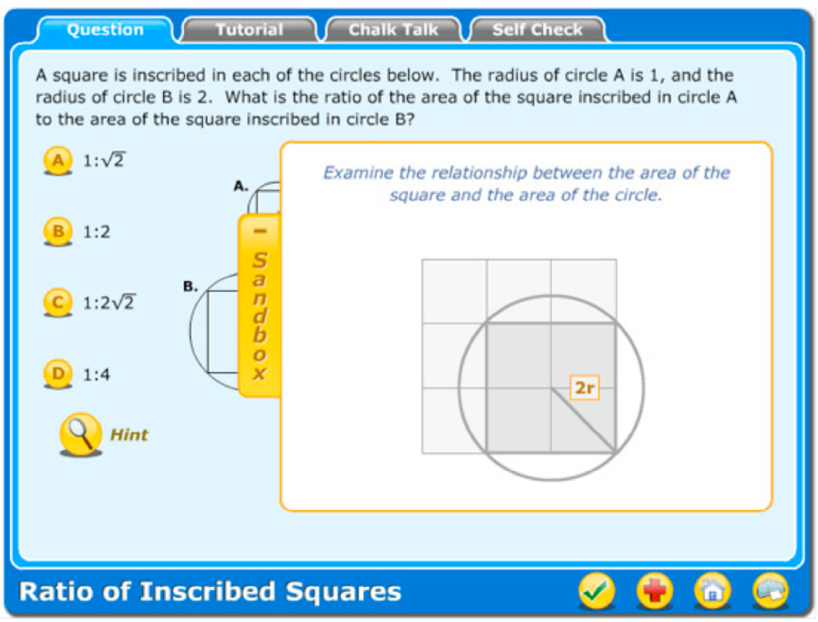

Figure 4: Sandbox Example

Sandbox: The Sandbox is a structured, graphical space where users can use customize tools play with the key variables and contexts of the problem. Providing a structured place for learners to play with a problem could trigger recall of the appropriate procedures involved. Sandbox design aimed at developing the right conceptual tools for hands-on learners to trigger recall

In the RMLO 6 (Logic of Negation), for example, the design team struggled to determine the appropriate representations that would allow for learners to play with implications of negating “All P is Q.” We decided on a 3-phase Venn diagram representation designed to show the set membership relation between P and Q. Figure 5 shows the initial condition: All P is Q. Learners can drag the circle that contains P in and out of the circle that contains Q to produce different propositional descriptions. These kinds of dynamic representations were designed to take advantage of the graphical medium and provide learners with manipulable tools to facilitate recall. Each Sandbox design converted the propositions in which the problem was framed into graphical elements that allowed for direct manipulation of salient problem variables. Other examples of sandbox design included visualizing different aspects of a 3-D cube in RMLO 2 (Area of a Cross Section) and the ability to manipulate a taxi to determine the relation of fare to time in RMLO 11 (Step Function).

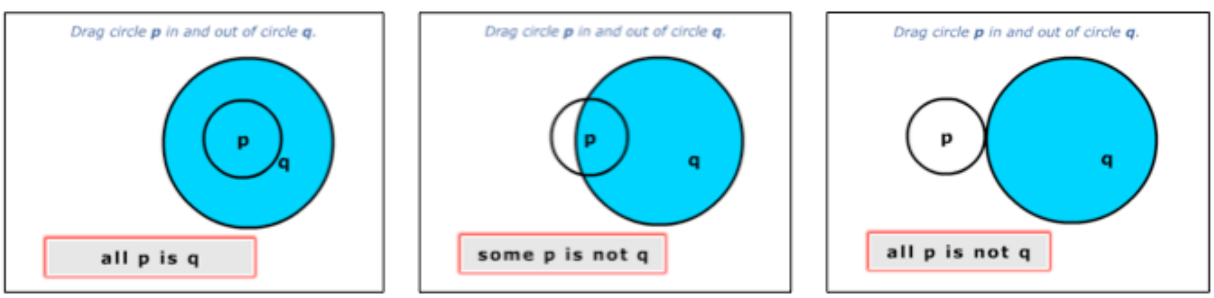

Figure 5: Logic of Negation Sandbox

Tutorials: The tutorial presents a stripped-down graphics-based instructional process designed to provide a cognitive walk through salient aspects of the problem. The tutorial was designed for learners who could not recognize or recall the problem type at all, but framed so that learners could recall the gist of the appropriate solution path without directly showing the answer to the problem. Tutorials represent a more comprehensive approach to reminding students not only which procedures are involved, but how the procedures should be used. The design of the tutorial typically occupied 40-50% of the total Learning Object development time. The tutorial design process was typically led by intelligent novices who used the math experts as consultants to the process of working through the solution strategies. Instead of providing a global approach to re-instruction, the tutorials were designed to begin exposition in the conceptual or procedural chokepoints that confused novices. Design team novices were challenged to explain just where conceptual and procedural confusions occurred, and to talk through the salient aspects of the problem that would need clarification in order to provide the solution.

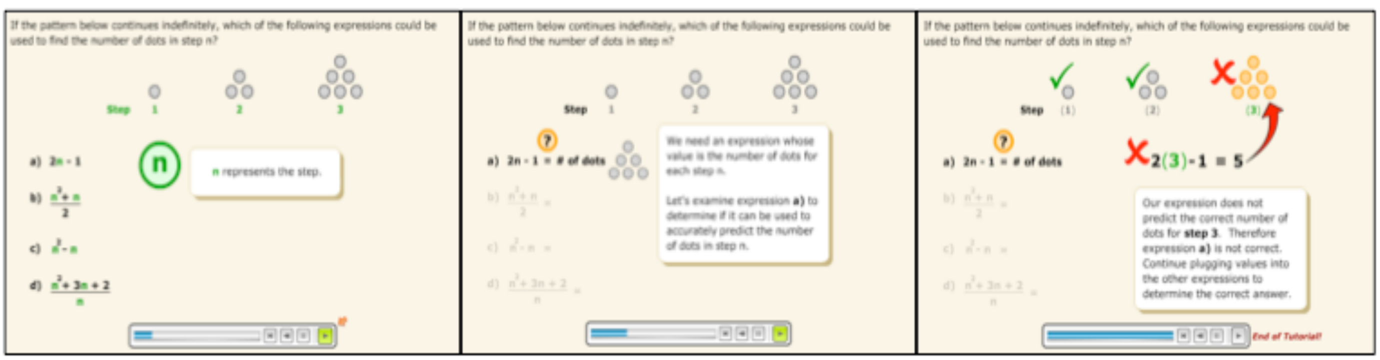

Figure 6: Pattern Recognition Tutorial

RMLO 7 (Pattern Recognition) provides a good example of the tutorial design process (Figure 6). Learners are asked to understand how to construct a variable to describe a pattern of additive growth ("If the pattern below continues indefinitely, which of the following expressions could be used to find the number of dots in step *n*?"). In the Hint section, we found that novices got stuck on the terminology of what counts as "*n*" in the problem – the number of the step or the number of dots? The tutorial begins with the reminder that *n* refers to the step number, then that the equations provided predict the number of dots. Once learners understand the relation between what counts as the variable and what the equations solve for, then determining the right answer is a matter of testing out which of the provided equations yields the correct answer.

The tutorial design process leaned heavily on standard test preparation techniques such as looking to the answers to determine the appropriate solution path, eliminating obviously misleading answers, and applying generic problem solving procedures. However, the design team approached the development of each learning object in terms of the specific requirements of each problem to address just how understanding breaks down and could be recovered though engagement with a learning object. As we will discuss later, this encounter with the specific bottlenecks of each kind of problem was not the most efficient approach to design, but it resulted in opportunities to deeply explore just how adult understanding breaks down in procedural recall, and to design tools targeted at addressing recall problems just where they happen.

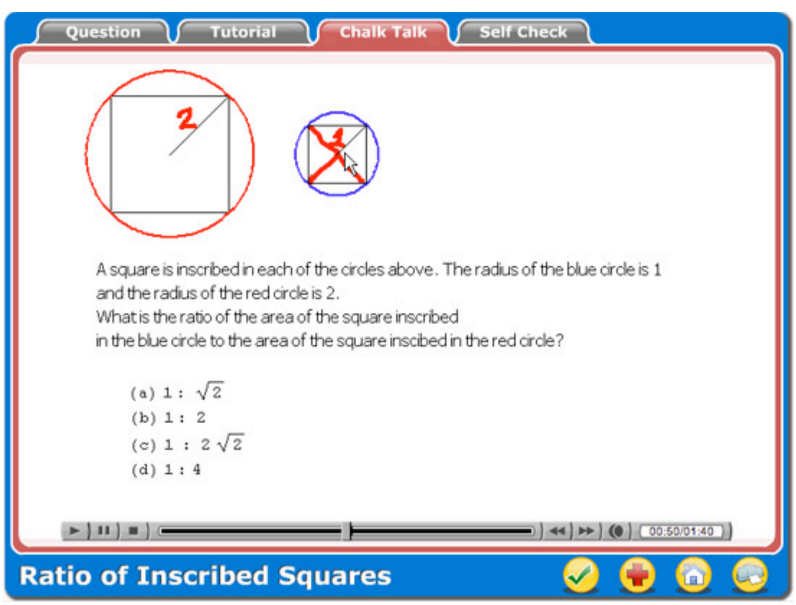

Figure 7: Chalk Talk

Chalk Talks (Figure 7): Chalk Talks provided the most comprehensive and traditional approach to sparking recall. A chalk talk session typically involved a math teacher talking through the problem-framing and solution process as if it were a classroom lesson. A situated cognition perspective (e.g. Clancey, 1997) suggests that the social and physical contexts in which new information is acquired plays a powerful role in recall. Scripts that are not rehearsed independent of the context in which they were learned, such as math lessons, are particularly vulnerable to restrictive association with a given time and place. The Chalk Talks were designed to virtually reconstruct the context of classroom presentation in which math lessons were originally experienced by many learners. The Chalk Talk design involved inviting a high-school or college math teacher to present a solution to the particular problem. These mini-lectures were captured through the *Camtasia* software and stored as Flash files within the learning objects. Each object included access to this more traditional path to recall.

Although the design process resulted in a durable, generative Flash-based template to support subsequent RMLO design, the real work involved determining just which kinds of support were appropriate for a given math problem, and with the struggle about how to graphically represent the path from breakdown in the context of a given learning object. We found that the development of content for each learning object took from 10-20 hours and typically involved more than two iterations through the design team and with groups of potential users. This attention to the specific learning requirements for each math problem pushed the design process from an example of technology design for e-learning to an investigation of how multi-media learning objects can help researchers explore math understanding. In the discussion section below we hope to reinforce our central finding that, although the template for the learning objects was important, careful attention to the design process gives us clear insights into how the design of learning objects serves as a instrument to expand our understanding of how learners can remember math.

3.3 Evaluating The Learning Objects

Did the Remembering Math Learning Objects actually help learners to remember dormant knowledge? We developed an on-line process to evaluate the effects of the learning objects consisting of a) a pre-question that was similar to the question in the learning object; b) exposure to the learning object; c) a post-question similar to the one provided in the learning object; d) an opportunity to reflect on the strengths and limitations of the learning object, and e) a set of questions about demographic characteristics of participants (Appendix 2). Each participant engaged with 6 of the learning object units during the assessment process.

We evaluated the effects of the learning objects with a sample of 59 adult learners in Fall of 2006. Using convenience sampling, we drew participants from a group of non-math specialist in-service and pre-service teachers enrolled in graduate and undergraduate education courses at the University of Wisconsin-Madison. The sample was comprised of 66% females and 34% males. The majority of the sample (53%) was between the ages of 18 to 24, while 37% were between the ages of 25 to 34, and the remaining 10% were between 35 and 50 years old. In terms of ethnicity, a majority of 86% identified themselves as White/Non-Hispanic, 5% as Hispanic/Latino, and 5% as Asian (with 4% self identifying as other). As for qualifications, 47% of the sample were pre-service teachers working on both their bachelor’s degree and certification in a field other than math, 7% held a BA and were seeking only certification, and the remaining 46% were seeking a graduate degree or additional certification (i.e. Administrative Licensure). Only one participant was teaching math while the data was being collected, and one additional participant had prior experience teaching math.

An initial analysis of the mean difference between pre and post test scores indicates that on average, test scores improved after exposure to the learning objects. Using an average of the 6 pre- and post-test scores for each participant, we subtracted the pre-test average from the post-test average to derive an average level of improvement for each participant. The mean average level of improvement for the sample was approximately 19% between pre- and post-test results with a standard deviation of 23.25%. In order to know if we could trust these findings, we conducted a one sample t-test to determine if the improvement between pre- and post-test was statistically significant. This resulted in a t-statistic of 6.2515 ($p \cong 0$). This demonstrates that in general the math learning objects improved the participant's ability to accurately complete the problems.

	N	Mean Pretest	Mean Post Test	Mean Difference	SD
Preservice	27	.40	.67	.27	5.908
Inservice	32	.59	.72	.13	3.215
Total	59	.50	.69	.19	6.252

Table 1: Descriptive statistics on the difference in scores, Preservice & Inservice Teachers

When these results are disaggregated by pre-service and in-service participants, we find that the learning objects actually had a more powerful effect for pre-service teachers. While in-service educators demonstrated a mean improvement of 13%, pre-service educators improved by an average of almost 27%. However, the in-service educators performed at a higher average level averaging 72% on the post-test, while the pre service educators only averaged 67% on the post test. To what extent greater improvement for in-service teachers is a factor of lower scores on the pre-test and therefore the possibility of greater improvement than pre-service teachers is unknown. This difference may also be attributable to sampling bias, as in-service teachers in the sample were self-selected by their participation in graduate studies. Even so, this finding warrants further investigation into whether the learning differences between the pre- and in-service groups is significant, and whether this indicates that the two groups have different needs in future RMLO design. While the level of improvement may represent adequate improvement for users who are at a failing level on the PRAXIS II exam to achieve passing, this might answer our immediate policy concern but fail to assure that these teachers have actual mastery over the skills in question. Interestingly, this may actually be a reflection of the design goal of these learning objects, as they were created around the near term objective of giving educators adequate familiarity with the mathematical concepts to pass the test, rather than being designed to inculcate deep conceptual understanding.

	Value of Learning Object		Ease of Use of Learning Object	
Total (354)	N	Rating	N	Rating
Hint	259 (73%)	4.65	247 (70%)	3.39
Sandbox	271 (77%)	4.38	265 (75%)	3.65
Tutorial	224 (63%)	4.71	213 (60%)	4.37
Chalk Talk	173 (49%)	4.63	157 (44%)	4.28

Table 2. Rating the value and ease of use of RMLOs.

The second part of the Learning Object evaluation process invited participants to rate their experience interacting with the RMLO system. The RMLO assessment process guided each participant through six of the twelve learning objects. Participants were then invited to rate the value and the ease of use of each reminding function of each learning object on a 1-5 scale, where 5 was the highest rating. Table 2 summarizes the overall rating results. The Total possible ratings is calculated from 59 RMLO users each exposed to 6 Learning Objects for a total of 354 possible ratings for the four learning object reminding functions across each participant's experience. N refers to the actual number of ratings included in the system for each reminding function of each learning object. We inferred from the overall findings that the Hint and Sandbox reminding functions were rated most often, but that the Tutorial and the Chalk Talks were seen as the most helpful and easiest to use reminding functions.

	In-Service Teachers				Pre-Service Teachers			
	LO Value		LO Ease of Use		LO Value		LO Ease of Use	
	N	Rating	N	Rating	N	Rating	N	Rating
Hint	149 (78%)	4.66	141 (73%)	3.35	110 (68%)	4.65	106 (65%)	3.43
Sandbox	151 (79%)	4.37	148 (77%)	3.46	120 (74%)	4.38	117 (72%)	3.49
Tutorial	142 (74%)	4.75	135 (70%)	4.33	82 (51%)	4.65	78 (48%)	4.42
Chalk Talk	117 (61%)	4.67	108 (56%)	4.34	56 (35%)	4.55	49 (30%)	4.14
	73%	4.61	69%	3.65	57%	4.55	54%	3.59

Table 3. Comparison of Pre-Service and In-Service teachers on Learning Object Reminding Function Value and Usability

The contrast between the ease of use ratings for the Hint and Sandbox on the one hand, and the Chalk Talk and Tutorial on the other, may result from the relative difficulties of the problems around which the RMLOs were built. Participants who struggled with a given learning object may have found the Hint and Sandbox more difficult to use because of the limited reminding capacity for each function. Such users would then find more the comprehensive explanations available in the Tutorial and the Chalk Talk to be a surer path to a given solution. However, the gap between the value and the ease of use of the Hint and Sandbox functions suggests that we need to investigate the relation between value and usability more carefully. Table 3 compares the ratings of pre-service and in-service teachers. Pre-service teachers rated a lower percentage of the learning objects, and gave slightly lower ratings than in-service teachers, but as seen in Table 1, seemed to learn more from the RMLOs than in-service teachers.

We invited participants to rate their confidence on the problem type on a 3-point scale both before and after using the learning objects. On average, subjects felt more confident in solving the problem types they encountered after utilizing the learning objects. Interestingly, this change in user confidence did not have a reportable correlation with improvements in user scores across learning objects. Our preliminary results indicate that use of the learning objects improved both the users' confidence in solving problem types and their capacity to do so. In addition to measuring users' confidence, we were also able to use participant ratings about the helpfulness of individual learning objects and the specific features of the objects to guide the iterative redesign process. On a 1-3 scale, with 3 being the most helpful, the participants rated the RMLOs at 2.45, with the pre-service teachers average at 2.37 and the in-service at 2.54. This feedback provided useful information about where the design was flawed and which problems were not communicating the intended learning goals.

It is important to note several limitations in the evaluation of learning gains described above. First, because we used convenience sampling, our sample of participants was voluntaristic rather than random. Second, we were not able to generate comparative outcome information on particular problems that provided the best learning gains, or particular features of problems that provided the most helpful recollection cues. Third, we invited the participants to engage with the learning objects without a sense of the context for which the objects would be deployed. Finally, and perhaps most crucially, if the learning objects were in fact utilized by the target population, they would probably supplement other test preparation activities. A full evaluation of the learning objects would produce more accurate information about the differential effects of the objects. Still, our initial efforts at evaluation confirm that there was a difference between pre- and post-tests in math problem performance and provide a useful direction for further evaluation in subsequent studies.

5.0 Discussion

The design of learning objects is promising as a development path, particularly for adult e-learning due to its flexibility and modularity. If there are lessons to be learned from the design of the RMLOs, they lie primarily in the work taken to ensure that the objects fit the needs of the targeted learners. Our focus on the design process allowed us to elicit hypotheses about where conceptual breakdowns occur in given math problems. We observed that the breakdown typically occurred when learners forgot the specific functions of specialized terminology. Concepts such as functions, variables, the rules of inference and the features of reading a scatter plot broke down when learners tried to use their uncertain understanding to reason through problems. Of course, the assessment designers sought to anticipate these typical misunderstandings in multiple choice answer options, and designed items to discern the subject's ability to understand conceptual nuance. Based on open-ended feedback from our subjects we found that some of our objects seemed to directly illuminate just the conceptual aspects required for successful solution, while others seemed to miss the mark and had little effect on subject learning. Still, the promise of developing learning objects that could steer between no instruction and a full-course of study is an appealing direction for reviving dormant knowledge in subject areas which most adult Americans have had extensive exposure.

There were several important issues that we need to address about this study. First, we emphasized the role of specific math problems in the design process. Textbook designers and classroom have long engaged in the selection and reconstruction of solution patterns as a pedagogical aid. We do not suggest that there is anything particularly novel in the way we developed solution paths for these math problems. However, we do emphasize that our focus on conceptual and procedural breakdowns of novices in the design process helped us to identify the conditions for successful recall in our intended audience. Instead of beginning with standards for what math teachers need to know, or adopting comprehensive approaches to reteaching math, or focusing on generic test-taking strategies to help learners "game" the test, our attention to specific problems was intended to ground recall in authentic contexts that could legitimately bridge remembered math to real occasions of problem-solving.

The focus on particular math problems also helped us to follow the guide of minimalist design. Carroll (1992) argued that since learners desire environments with meaningful context and goals. We felt that focusing the design of the RMLOs on the solution of particular, difficult problems would provide a meaningful context for the Remembering Math learners, and that our interface should organize all learning aids to feature this central learning context. Similarly, Carroll argued that the interface should rely on the learner's errors and recovery processes to facilitate the learning process. Focusing on specific problems allowed the RMLOs to provide Sandbox and Tutorial activities designed to exploit the hypothesis and testing activities learners would use to reconstruct past math knowledge. These types of design activities and rationales may well characterize the typical problem-development activities of learning object designers and textbook writers. Our decision to highlight the breakdown elicitation process would then bring to the fore a critical aspect of design practice in order to guide subsequent learning object design.

A second issue with our study was the discussion of the activity of one design team and the development and testing of 12 RMLOs. We may well have assembled an idiosyncratic team composed of novices with a peculiar sense of what constituted an obstacle to recall. Although we tried to temper the possible idiosyncrasy of the design team composition with a collaborative, iterative design process, to ensure the value of the design process we would need to a) replicate the design process with other team members, and b) conduct more systematic evaluation to ascertain just which learning object features best trigger recall with which kinds of learners. Interestingly, the University of Wisconsin System used the Remembering Math templates and design process to develop RMLOs with design teams across the state in 2006-08. Working with the design teams involved in this broader development process might allow us to see how (or whether) the design process we specify here would result in similar RMLOs or similar outcomes when implemented at scale.

Finally, we consider the question of whether the RMLOs provide an appropriate model for adult learning. The intentionally narrow scope of each learning object restricts the intended learning outcome to recalling the specific procedural or conceptual knowledge required to solve a particular problem. These objects would provide no substitute for a more comprehensive approach to math education that helps learners acquire mathematical ways of knowing or flexible problem-solving skills. However, the restricted scope of the RMLOs has several advantages for the design and implementation of adult learning curricula. For example, focusing on specific problems allowed designers to understand just how obscure and forgotten definitions of mathematical terms, for example, obstructed the problem-solving process of adult learners. Clearing up procedural details through scaffolded recollection cues may well allow experienced learners with a quick path to reviving lost knowledge instead of relearning old content from scratch.

Of course, it might be noted that the majority of adult learners never really learned the basic math they are being challenged to recall in the first place, and that no amount of recollection will recover what was never there. These teachers, one might argue, should not be teaching math to anyone, and the testing process is well designed to remove their influence from the math classroom. Well-designed learning objects will obviously not spark the recovery of non-existent knowledge, but the design and the implementation of the RMLOs helped many teachers bridge the gap between disconnected and useful mathematical knowledge. Learning tools that can provide a nudge toward recovery of previously disconnected knowledge could help these teachers, who may be effective in other areas of instruction, to once again engage students in mathematical instruction.

6.0 Conclusion

As e-learning becomes increasingly ubiquitous, the need for models of instructional design that fit the specific contexts and learners in question becomes increasingly important. In the Remembering Math project, we confronted the learning needs of adult learners who required learning tools to help them recall mathematical skills and concepts they had previously forgotten, in a context that was not necessarily tied to a specific virtual or physical classroom setting. Towards this end, we established an original instructional design process for developing learning objects that would leverage digital settings to provide four unique modes of access to math concepts for adult learners whose individual needs were based on their ability to recall different parts of the math they needed to relearn.

This paper provides an example of what Chris Hoadley (2002) called a "design narrative." Design research has recently emerged in education research as a path for building practical tools (such as curricula, assessments, and teacher support tools) by engaging practitioners in collaborative design and assessment activities (Collins et al, 2004; Cobb, et. al., 2003). Building and testing learning objects would provide us with both a valuable form of user testing and program refinement, as well as granting insight into the conceptual breakdown and recovery practices of adult learners. However, recent discussions of design research (e.g. Design-Based Research Collective, 2003; Shavelson, et. al. 2003) have focused on the degree to which design research can inform conventional approaches to educational research. Following Hoadley, we suggest that a key contribution of design research is to articulate the rationale for building an intervention so that others may replicate the complex process of artifact building. Our approach here was develop a theory of action appropriate for helping teachers to remember once-learned math, then to describe how the theory of action was operationalized in a series of tools designed to produce the desired learning. The design narrative we have assembled has the dual function of making a contribution to e-learning research, and also making practical suggestions for how designers to build new kinds of objects for adult learners.

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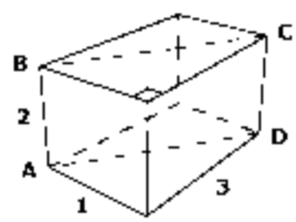
Appendix 1: RMLO Problems (with Hints)

1. *Adding Fractions*: If bd ≠ 0, then a/b + c/d = ?

   Hint: Adding fractions requires a common denominator

2. *Area of a Cross Section*: A rectangular solid with dimensions 1, 2, and 3 is shown below. What is the area of the Cross Section ABCD?

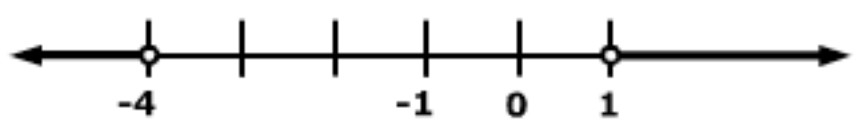
   a. 

   Hint: The diagonal of a rectangle is the hypotenuse of a triangle

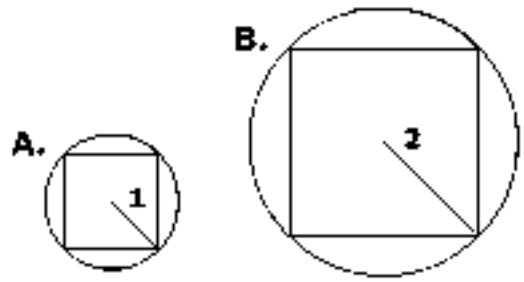
3. *Fraction Conversions*: A certain recipe calls for ¼ cup of white sugar, ½ cup of brown sugar, 2 ¼ cups of flour, ¼ teaspoon of salt, and two tablespoons of cornstarch. The amount of cornstarch called for is approximately what fraction of the total amount of sugar called for? Hint: How much total sugar is there?

4. *Inequalities*: The graph shown on the number line below represents the set of values of x satisfying which of the following inequalities?

   Hint: The endpoints are key to defining the factors



5. *Ratio of Inscribed Squares*: A square is inscribed in each of the circles below. The radius of the circle B is 2. What is the ratio of the area of the square inscribed in Circle A to the area of the square inscribed in circle b?

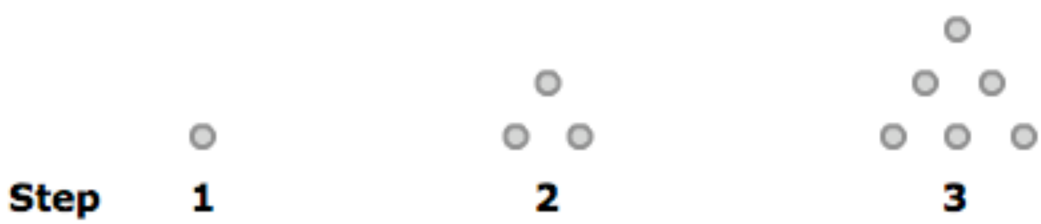


Hint: Doubling the radius quadruples the area

6. *Logic of Negation*: What is the negation of the statement "All P is Q"? a) some p is q; b) some p is not q; c) If it is not q, it is p; d) If it is not q, it is not p.

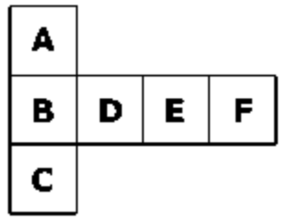
   Hint: What would make this statement false?

7. *Pattern Recognition*: If the pattern below continues indefinitely, which of the following expressions could be used to find the number of dots in step n?



   Hint: *n* represents the step; # *of dots* is the value of the expression

8. *Probability*: The T-shaped figure below consists of six congruent squares. A coin will be placed randomly on one of the squares. It will then be moves randomly to a square adjacent to the square on which it was originally placed. What is the probability that, after the coin has been moved, it will be on square E?



Hint: How many squares are adjacent to D? B?

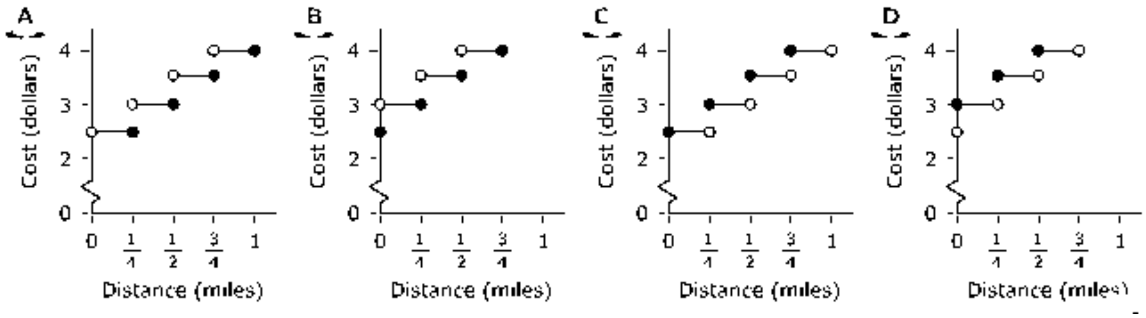
9. *Slope and Perpendicular Lines*: Which of the following is an equation of a line that is perpendicular to the line: y=2x+3? A) y=-2x+6; b) y=1/2x+3; c) y= 1/2x+3; d) y=2x+6. Hint: Slopes of perpendicular lines are negative reciprocals

10. *Stem and Leaf Plot*: The table below is a stem-and-leaf plot of the weights of 25 members of a men's fitness club. What percent of the club members weigh less than 155 lbs?

Weights (pounds)	
Stem	Leaf
12	0, 2
13	1, 3, 3
14	2, 4, 5, 8
15	0, 1, 4, 7, 7, 9
16	2, 8, 8
17	1, 6, 9
18	0, 0, 4
19	5

Hint: Each leaf represents the weight of one member.

11. *Step Functions*: A taxi ride costs $2.50 for the first ¼ mile or fraction thereof, plus $0.50 for each additional ¼ mile or fraction thereof. Which of the following graphs represent the total cost of the ride as a function of distance travelled?



Hint: Open circles mean that the enclosed point is not included on the line.

12. *Proportional Reasoning*: According to a survey of 100 students, 73 students took a math course and 57 took a music course. Of those surveyed, 22 reportedly took a math course but not a music course. How many students took neither a music course nor a math course? Hint: How many people took music? How many took math alone?

Appendix 2: RMLO Assessment Process

1) Please examine the question to the right,

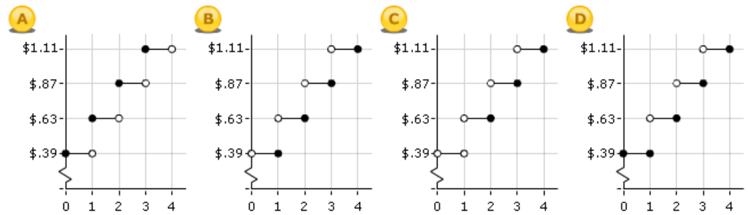
and
select the correct answer below:

a. ◯
b. ◯
c. ◯
d. ◯

How confident are you that you got the answer to this question correct?

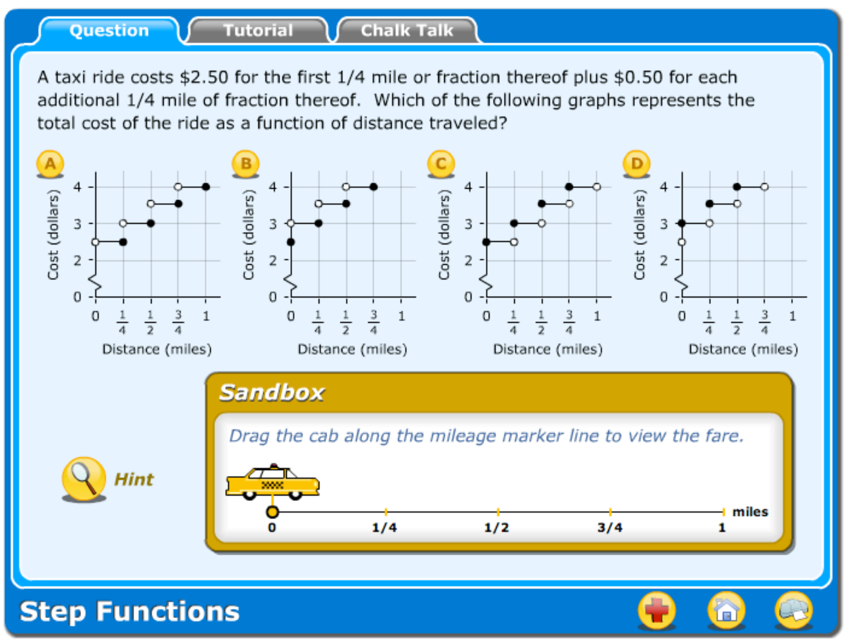
◯ Very Confident
◯ Somewhat Confident
◯ Not Confident

The cost to mail a first class package is 39 cents for the first ounce and 24 cents for each additional ounce. Which of the graphs below represents the cost to mail packages of various weights?



1	2	3	4	5	6	D	Continue >>

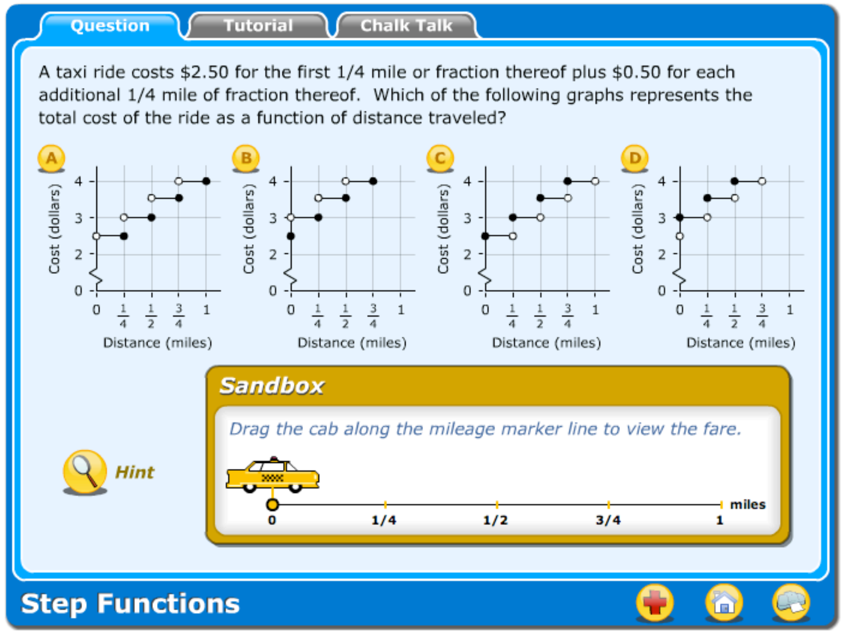
2) Use the different parts of the learning object to explore step functions



1	2	3	4	5	6	D	Continue >>

3) Please rate the **value** of each feature for your learning
(1: very little value - 5: extremely valuable,
NU: not used)

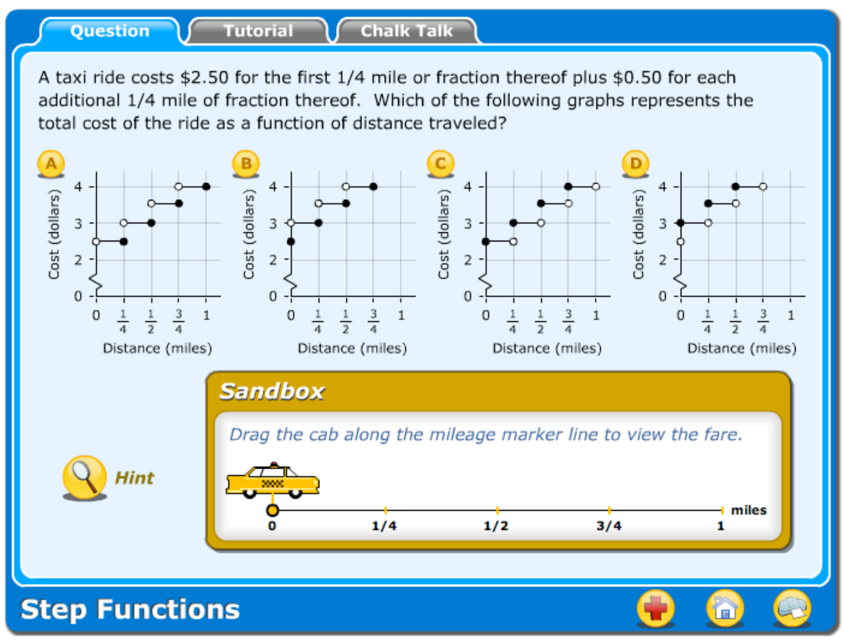
	NU	1	2	3	4	5
Hint	○	○	○	○	○	○
Sandbox	○	○	○	○	○	○
Tutorial	○	○	○	○	○	○
ChalkTalk	○	○	○	○	○	○



1 2 3 4 5 6 D Continue >>

4) Please rate **how easy** it was to use each feature:
(1: extremely difficult - 5: very easy,
NU: not used)

	NU	1	2	3	4	5
Hint	○	○	○	○	○	○
Sandbox	○	○	○	○	○	○
Tutorial	○	○	○	○	○	○
ChalkTalk	○	○	○	○	○	○

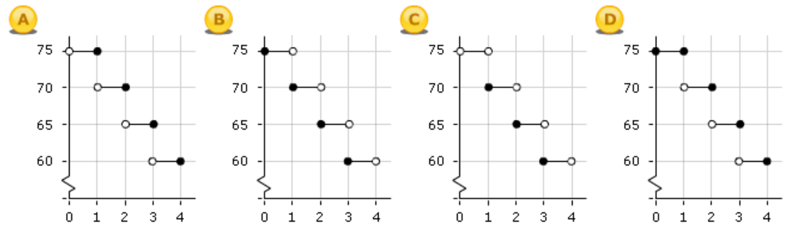


1 2 3 4 5 6 D Continue >>

5) Please examine the question to the right,

and
select the correct answer below. Please note that the question is slightly different than the question in the learning object:

The price of a article listed on eBay is $75 for the first day and decreases by $5 for each day until it is sold? Which of the graphs below represents its price for a few days if left unsold?



a. ◯
b. ◯
c. ◯
d. ◯

How confident are you that you got the answer to this question correct?

◯ Very Confident
◯ Somewhat Confident
◯ Not Confident

6) Did you find the learning object helpful for answering the question?

◯ Yes
◯ Somewhat
◯ Not Helpful

Please provide any additional comments

By pressing "Save & Exit" you can return at a later time to finish the survey.

1 | 2 | 3 | 4 | 5 | 6 | D | Save & Exit | Next Module >>